Maximum-norm error estimate of the finite volume approximation for a convection-diffusion equation

Norikazu Saito
Graduate School of Mathematical Sciences,The University of Tokyo,
3-8-1 Komaba, Meguro, Tokyo, 153-8914 Japan
norikazu@ms.u-tokyo.ac.jp

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We consider the finite volume (FV) approximation in the sense of [1] for a convection-diffusion equation for $u = u(x,t)$ of $\overline{\Omega} \times [0,T]$,

$$\begin{cases} u_t - \nabla \cdot (\nabla u - bu) = f \quad \text{in} \quad \Omega \times (0,T), \\ u = g \quad \text{on} \quad \partial\Omega \times (0,T), \quad u|_{t=0} = u_0 \quad \text{on} \quad \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^2$ denotes a polygonal domain in $\mathbb{R}^2$; T a positive constant; $b : \Omega \times (0,T) \to \mathbb{R}^2$, $f : \Omega \times (0,T) \to \mathbb{R}$, $g : \partial\Omega \times (0,T) \to \mathbb{R}$ and $u_0 : \Omega \to \mathbb{R}$ are given smooth functions.

The purpose of this paper is to prove an error estimate in L^∞ norm, under some appropriate assumptions,

$$\|u - u_h^n\|_{L^\infty(0,T;\Omega)} = O(h + \Delta t) \quad (h \downarrow 0),$$

where u_h^n denotes the FV approximation under consideration with the space and time discretization parameters h and Δt. To prove this uniform convergence result, we basically follow the method of Tabata [2]. In fact, one of the crucial points of Tabata's method is that FV schemes are treated as finite difference schemes defined on irregular meshes, and it can be directly applied to analysis of FV schemes defined on admissible meshes.

References

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[2] M. Tabata: *Uniform convergence of the upwind finite element approximation for semilinear parabolic problems*, J. Math. Kyoto Univ. **18** (1978) 327–351.