AN IMPLICIT FINITE DIFFERENCE SCHEME FOR FOCUSING SOLUTIONS OF THE GENERALIZED DAVEY-STEWARTSON SYSTEM

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The generalized Davey-Stewartson (GDS) system is given by

$$\begin{aligned} &i\,u_t + \sigma u_{xx} + u_{yy} = \kappa \mid u \mid^2 u + \gamma(\varphi_{1,x} + \varphi_{2,y})u \\ &\varphi_{1,xx} + m_2\varphi_{1,yy} + n\varphi_{2,xy} = (\mid u \mid^2)_x \\ &\lambda\varphi_{2,xx} + m_1\varphi_{2,yy} + n\varphi_{1,xy} = (\mid u \mid^2)_y \end{aligned}$$

where u and φ_1, φ_2 are, respectively, the complex- and the real-valued functions of spatial coordinates x, y and the time t. The parameters $\sigma, \kappa, \gamma, m_1, m_2, \lambda, n$ are real constants and σ is normalized as $\mid \sigma \mid = 1$. The GDS system has been derived to model $2 + 1$ dimensional wave propagation in a bulk medium composed of an elastic material with couple stresses [1]. The parametric relation $(\lambda - 1)(m_2 - m_1) = n^2$ follows from the structure of the physical constants and plays a key role in the analysis of these equations. The GDS system is classified according to the signs of parameters $(\sigma, m_1, m_2, \lambda)$. In this study, we consider (+,+,+,+) elliptic-elliptic-elliptic (EEE) and (–,+,+,+) hyperbolic-elliptic-elliptic (HEE) cases.

In this study, the generalized Davey-Stewartson (GDS) system is solved by a numerical method which is based on an extension of the relaxation method introduced in [2]. In the hyperbolic-elliptic-elliptic case, we numerically test the relaxation method by using the analytical blow-up profile. In the elliptic-elliptic-elliptic case, we compare the numerical results with the analytical global existence and blow-up results [3] for certain ranges of parameters. Numerical tests show that the relaxation method does not miss the blow-up phenomena and provides accurate results for the GDS system.

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