Bergman Orthogonal Polynomials: Asymptotics, Zeros and Shape Reconstruction

Nikos Stylianopoulos

Department of Mathematics and Statistics,

University of Cyprus,

P.O. Box 20537, CY 1678 Nicosia, Cyprus

nikos@ucy.ac.cy

http://www.ucy.ac.cy/nikos.html

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Let $G := \cup_{j=1}^{N} G_j$ be the union of N mutually exterior, bounded Jordan domains G_j in the complex plane and let $\{P_n\}_{n=0}^{\infty}$ denote the sequence of *Bergman polynomials* of G. This is defined as the sequence

$$P_n(z) = \lambda_n z^n + \cdots, \quad \lambda_n > 0, \quad n = 0, 1, 2, \ldots,$$

of polynomials that are orthonormal with respect to the inner product

$$\langle f, g \rangle := \int_G f(z)\overline{g(z)} dA(z),$$

where dA stands for the area measure. (In the case when $N > 1$ we call G an *archipelago*.)

The purpose of the talk is to present some very recent developments regarding the theory and applications of Bergman polynomials.

These developments include: (i) Strong asymptotics for the Bergman polynomials, in the single component case $N = 1$, over domains with corners. (ii) Distribution of zeros and estimates of $P_n(z)$, in the archipelago case. (iii) An reconstruction algorithm for recovering the shape of an archipelago G from a finite section of its complex moment matrix:

$$\mu_{mn} := \int_G z^m \overline{z}^n dA(z), \;\; 0 \leq m, n \leq k.$$

The importance of this algorithm in the general shape-reconstruction-problem in 2D geometric tomography, is underlined by the fact that suitable tomographic data (for example, parallel ray measurements) of the uniformly distributed area measure on G, can be transformed into a finite section of the moment matrix.