Numerical Solution of Discontinuous Differential Systems:
Approaching the Discontinuity Surface from One Side

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We present a numerical approach to treat discontinuous differential systems of ODEs of the type:
\[ x' = f_1(x) \text{ when } h(x) < 0 \quad \text{and} \quad x' = f_2(x) \text{ when } h(x) > 0, \]
where \( \Sigma := \{ x : h(x) = 0 \} \) is a smooth co-dimension one discontinuity surface, and with \( f_1 \neq f_2 \) for \( x \in \Sigma \). Often, \( f_1 \) and \( f_2 \) are defined on the whole space, but there are applications where \( f_1 \) is not defined above \( \Sigma \) and \( f_2 \) is not defined below \( \Sigma \). For this reason, we consider numerical schemes –based on a class of explicit Runge-Kutta methods– which do not evaluate \( f_1 \) above \( \Sigma \) (respectively, \( f_2 \) below \( \Sigma \)).