Numerical Solution of Discontinuous Differential Systems: Approaching the Discontinuity Surface from One Side

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We present a numerical approach to treat discontinuous differential systems of ODEs of the type: $x' = f_1(x)$ when $h(x) < 0$ and $x' = f_2(x)$ when $h(x) > 0$, where $\Sigma := \{x : \ h(x) = 0\}$ is a smooth co-dimension one discontinuity surface, and with $f_1 \neq f_2$ for $x \in \Sigma$. Often, f_1 and f_2 are defined on the whole space, but there are applications where f_1 is not defined above Σ and f_2 is not defined below Σ. For this reason, we consider numerical schemes –based on a class of explicit Runge-Kutta methods– which do not evaluate f_1 above Σ (respectively, f_2 below Σ).