High-order accurate numerical pressure correction based on Geometric MultiGrid schemes for the incompressible Navier-Stokes equations

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Industrial and environmental applications of computational fluid mechanics, such as wind turbine blades, high-speed trains, naval transportation, aircraft wings, simulation of blood and atmospheric flow, require detail flowfield information. High order numerical methods offer higher resolution, over low accuracy schemes, for the numerical solution of the incompressible Navier-Stokes equations, for example by marching the solution in time with Runge-Kutta methods and discretizing in space by fourth-order compact difference schemes. The same order of accuracy should be maintained for the pressure correction applied to ensure incompressibility, a procedure which amounts in solving a Poisson-type equation. This is a highly demanding procedure, regarding computational cost, of the overall numerical method. The linear system to be solved is general, large and sparse, suggesting the use of iterative solvers to save computational cost. Estimation of the eigenvalues of the coefficient matrix showed that preconditioned iterative methods (specifically BiConjugate Gradient type methods with Jacobi preconditioning) converge faster than other classical solvers. Nonetheless, execution time for the solution of the linear system remains a forbidding factor for realistic applications. An improved performance is obtained using Schur Complement type iterations, but a significant convergence acceleration is achieved with the incorporation of geometric multigrid techniques into the iterative solver. The performance of several multigrid algorithms is investigated. The accuracy of the fourth-order compact difference scheme is not deteriorated by the incorporation of the multigrid technique, though realistic boundary conditions are effectively handled. The increase of scalability of the computation and the direct realization of the parallelism is ensured therefore, for the costly incompressibility problem. Multigrid schemes can accelerate the iterative solution process significantly (hundreds of times) even for fine discretizations.

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