NON-POLYNOMIAL SEXTIC SPLINE APPROACH FOR SOLVING VARIABLE COEFFIECIENT FOURTH-ORDER PARABOLIC EQUATIONS

Arshad Khan and Pooja Khandelwal

Department of Mathematics, Jamia Millia Islamia, New Delhi, INDIA.

Abstract: In this paper fourth-order variable coefficient parabolic partial differential equation, that governs the behaviour of a vibrating beam, is solved by using a three level implicit method based on non-polynomial sextic spline in space and finite difference discretization in time. We also obtain a new high accuracy schemes of $O(k^4 + h^8)$.
The linear stability of the presented methods is investigated. We solve test problems numerically to validate the derived methods. Numerical comparison with other existing methods shows the superiority of the presented scheme. We Consider the undamped transverse vibrations of a flexible straight beam whose supports do not contribute to the strain energy of the system and is represented by,

$$\mu(x)\frac{\partial^2 u}{\partial t^2} + \eta(x)\frac{\partial^4 u}{\partial x^4} = f(x,t),\ L_0 \leq x \leq L_1, t > 0, \tag{1}$$

subject to the initial conditions $u(x,0) = g_0(x)$ and $u_t(x,0) = g_1(x)$, for $L_0 \leq x \leq L_1$, and with boundary conditions

$$u(L_0,t) = f_0(t),\ u(L_1,t) = f_1(t) \ \text{ and } \ u_{xx}(L_0,t) = p_0(t), u_{xx}(L_1,t) = p_1(t),\ t \geq 0$$

where u is the transverse displacement of the beam,$\mu(x) > 0$ and $\eta(x) > 0$ are mass per unit length and beam bending stiffness, t and x are time and distance variables respectively, $f(x,t)$ is dynamic driving force per unit mass.