A stabilized GPBiCG method with a strategy to remedy accuracy of Bi-CG coefficients for solving linear systems

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Bi-conjugate gradient (Bi-CG) is a well-known method for solving linear systems $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$, where $\mathbf{A}$ is a given n-by-n matrix, and $\boldsymbol{b}$ a given n-vector. A number of hybrid Bi-CG methods such as conjugate gradient squared (CGS), Bi-CG STABilized (Bi-CGSTAB), BiCGStab2, BiCGstab(ℓ), Generalized Product-type Bi-CG (GPBiCG), and BiCG×MR2 have been developed to improve the convergence of Bi-CG and to avoid multiplication by the transpose conjugate $\mathbf{A}^*$ of $\mathbf{A}$.

The residual polynomials of hybrid Bi-CG methods can be expressed as the product of the bi-Lanczos polynomial and other, so-called *stabilizing polynomials*, say P_k, with $P_k(0) = 1$ and degree k. In GPBiCG and BiCG×MR2, these stabilizing polynomials can be generated by a three-term recurrence similar to the Lanczos polynomials but with different recurrence coefficients. As the Lanczos polynomials, they can also be obtained by coupled two-term recurrences. GPBiCG exploits the coupled two-term version, while BiCG×MR2 exploits the three-term variant. A third variant of GPBiCG, which is referred to as the RG variant of GPBiCG, has been proposed by Röllin and Gutknecht to improve the stability of the original GPBiCG method. The stabilizing polynomials of this variant also rely on the coupled two-term recurrences.

As in BiCG×MR2, we propose to redesign the recurrences of GPBiCG by coupling the coupled two-term recurrences of Bi-CG with the three-term recurrence for the stabilizing. However, the coupled two-term recurrences of Bi-CG that we use differ slightly from the standard ones. This modification appears to lead to slightly more accurate Bi-CG coefficients and allows a more elegant derivation of GPBiCG. We consider two combinations. The recurrences of the resulting two algorithms are different from those of the original GPBiCG, BiCG×MR2, and the RG variant of GPBiCG. The better accuracy of the Bi-CG coefficients in our new variants appears to be important specifically in cases of a long stagnation phase. We therefore propose a strategy to remedy accuracy of BiCG coefficients in our variants. Our numerical experiments show that our new variants are less affected by rounding errors than the original GPBiCG, the RG variant of GPBiCG and BiCG×MR2, and the stabilized GPBiCG method with the strategy is effective.