A Local Discontinuous Galerkin Scheme for the Nonlinear Parabolic -p-Laplace Type Equation

Ioannis Toulopoulos[a],

[a]Abteilung fuer Angewandte Mathematik Universitaet,

Freiburg, Deutschland

ioannis.toulopoulos@math.uni-freiburg.de,

Key words: parabolic p-Laplacian, Local discontinuous Galerkin, stability analysis,

In this work a local discontinuous Galerkin (LDG) scheme is presented for the approximation of the solution u of the following p-Laplacian problem:
let Ω be a bounded domain in $\mathbb{R}^d,\ d = 1, 2, 3$ and let $(0, T]$ be the time interval, find u such that

$$\begin{aligned} u_t - \nabla \cdot (|\nabla u|^{p-2}\nabla u) &= f \ in\ \Omega \times (0,T] &(1)\\ u_0(x) &= u(x,0), &(2)\\ (|\nabla u|^{p-2}\nabla u)\cdot \mathbf{n} &= u_N \ on\ \Gamma_N \times (0,T] &(3)\\ u &= u_D \ on\ \Gamma_D \times (0,T], &(4)\end{aligned}$$

where $|.| : \mathbb{R}^d \to \mathbb{R}$ is the Euclidean norm, $\partial\Omega = \Gamma_N + \Gamma_D$, $\mathbf{n}$ is the outward normal vector to $\partial\Omega$, $p \geq 2$ and the functions $f : \Omega \times (0, T] \to \mathbb{R}$, $u_0 : \Omega \to \mathbb{R}$, $u_N : \Gamma_N \times (0, T] \to \mathbb{R}$, $u_D : \Gamma_D \times (0, T] \to \mathbb{R}$ are given.

Classical finite element methods for the problem (1) have been proposed by many authors, see e.g. [1]. The last decade many DG methods have extensively presented for the numerical solution of hyperbolic type problems, but effective DG methods for the numerical simulation of general (more realistic) problems as (1) have not presented. The purpose of this work is in this direction. LDG methods was introduced by Cockburn and Shu [2]. The basic idea of LDG method is to rewrite the parabolic equation as a first order system of equations and to solve for u and $\nabla u = \mathbf{q}$ as independent unknowns. A particular feature of LDG methods is that u and $\mathbf{q}$ are approximated in the same degree polynomial space. For the semi-discrete approximate solution we are going to present a-priori bounds (stability bounds) in case of general boundary conditions as well as numerical solutions for classical benchmark problems.

References

[1] Barrett W. J. and Liu W. B. Finite element approximation of the parabolic p-laplacian. *SIAM J. Numer. Anal*, 31(2):413–428, 1994.

[2] Cockburn B. and Shu C.W. The local discontinuous Galerkin method for time dependent convectiondiffusion systems,. *SIAM J. Numer. Anal*, 6(35):24402463, 1998.