Application of variational iteration method and homotopy perturbation method to the Klein-Gordon-Schrödinger equation

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In this paper, we shall consider the coupled nonlinear Klein-Gordon-Schrödinger (K-G-S) equations in the form:

$$u_{tt} - c^2 u_{xx} + u + |v|^2 = 0,$$
$$iv_t + v_{xx} + uv = 0.$$

Here, u represents a complex scalar nucleon field and v a real scalar meson field.

Homotopy Perturbation Method (HPM) and Variational Iteration Method (VIM) are implemented for solving the K-G-S equation. The results were compared with results of Adomian decomposition Method (ADM). The results reveal that the HPM and VIM are very effective, convenient and quite accurate to systems of nonlinear partial differential equations.

Basic idea of the VIM: To illustrate the basic concepts of the VIM, we consider

$$Lu + Nu = g(x) \tag{1}$$

where L is a linear differential operator, N a nonlinear analytic operator, and $g(x)$ an inhomogeneous term. According to the VIM, we can construct a correction functional as follows:
$u_{n+1} = u_n + \int_0^x \lambda u_n(s) + N\widetilde{u}_n(s) - g(s)ds,$
where λ is a general Lagrange multiplier, which can be identify optimally via the variational theory, $\widetilde{u}_n$ is considered as a restricted variation , i.e.,$\delta\widetilde{u}_n 0$.

Basic idea of the HPM: for solving eq. (1) by HPM we can constract a homotopy in the form

$$H(v,p) = (1-p)(L(v) - L(u_0)) + p[Lv + Nv - g(x)] = 0, \quad p \in [0,\ 1] \tag{2}$$

where p is an embedding parameter, u_0 is an initial approximation of the Eq. (1). In HPM, one can use the embedding parameter as a small parameter. Therefore, the solution of Eq. (2) can be written as a power series of p in the form, $v = v_0 + pv_1 + p^2 v_2 + ...$ By setting $p = 1$, one can get an approximate solution of the Eq. (1) as, $u = \lim_{p\to 1} v_0 + v_1 + v_2 + ...$

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