On correct boundary conditions in numerical schemes for the gravity wave equations

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In the atmosphere-ocean regional models, the initial conditions are supplied by data assimilation schemes and the boundary conditions are usually provided by a global model. Therefore, the values of almost all the prognostic fields are usually available on the domain boundary and there are many options for specification of the boundary conditions. The main difficulties for the formulation of the differential conditions at artificial boundaries are usually related to the necessity to mimic the physical medium, which surrounds the problem domain, and to specify physically reasonable conditions, which produce a well-posed mathematical problem. In discrete problems, besides keeping the well-posedness of differential problems, other important issues are the accuracy of approximation and the numerical stability, which connect differential and discrete problems. Different approaches to solution of these problems in atmosphere-ocean modeling can be found in a number of sources.

In this study we consider a non-physical growth of solutions leading to numerical blow-up in the cases when the choice of the boundary conditions appears to be physically justifiable and the initial-boundary value problem for the primitive differential system is well posed. This growth can be observed in both conditionally and absolutely stable numerical schemes and it can not be eliminated by reducing the time step. Since the corresponding numerical problem with the periodic boundary conditions is conditionally or absolutely stable, such instability is related to the specific non-periodic boundary conditions. We study this problem in a simplified model of one-dimensional gravity waves, which allows us to perform complete theoretical analysis and reveal the causes of such instability. Based on the performed analysis and numerical experiments, some recommendations for choosing the boundary conditions are given to avoid this non-physical behavior of numerical solutions.