The construction of second derivative general linear methods for numerical solution of ODEs

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Second derivative general linear methods (SGLMs) are an extension of general linear methods (GLMs) that are used for the numerical solution of autonomous system of ordinary differential equations

$$y' = f(y(x)), \qquad y : \mathbb{R} \to \mathbb{R}^m, \qquad f : \mathbb{R}^m \to \mathbb{R}^m. \tag{1}$$

An SGLM makes use of r input and output values and s first and second derivatives stage values. These methods are characterized by six matrices denoted by $A, \overline{A} \in \mathbb{R}^{s\times s}$, $U \in \mathbb{R}^{s\times r}$, $B, \overline{B} \in \mathbb{R}^{r\times s}$ and $V \in \mathbb{R}^{r\times r}$. In an SGLM, these quantities are related by the formula

$$Y_i^{[n]} = \sum_{j=1}^{s} h a_{ij} f(Y_j^{[n]}) + \sum_{j=1}^{s} h^2 \overline{a}_{ij} g(Y_j^{[n]}) + \sum_{j=1}^{r} u_{ij} y_j^{[n-1]}, \quad i = 1, 2, \cdots, s, \tag{2}$$

$$y_i^{[n]} = \sum_{j=1}^{s} h b_{ij} f(Y_j^{[n]}) + \sum_{j=1}^{s} h^2 \overline{b}_{ij} g(Y_j^{[n]}) + \sum_{j=1}^{r} v_{ij} y_j^{[n-1]}, \quad i = 1, 2, \cdots, r, \tag{3}$$

where $g(\cdot) = f'(\cdot)f(\cdot)$ for function f introduced in (1)

We will divide SGLMs into four types, depending on the nature of the differential system to be solved and the computer architecture that is used to implement these methods. For type 1, matrices A and $\overline{A}$ are lower triangular with the same element 0 on the diagonal and for type 2, matrices A and $\overline{A}$ are lower triangular with the same element $\lambda > 0$, $\mu < 0$ on the diagonal, respectively. Such methods are appropriate for nonstiff or stiff differential systems in a sequential computing environment. For type 3 or 4 methods, matrices A and $\overline{A}$ take the form

$$A = diag(\lambda, \lambda, \cdots, \lambda) = \lambda I, \tag{4}$$

$$\overline{A} = diag(\mu, \mu, \cdots, \mu) = \mu I, \tag{5}$$

where $\lambda = \mu = 0$ and $\lambda > 0$, $\mu < 0$, respectively. Such methods are appropriate for nonstiff or stiff differential systems in a parallel computing environment.

Since Runge–Kutta methods have excellent stability properties, it is desirable that SGLMs to be equipped by Runge–Kutta stability property. In this paper, we will construct some methods in type 4 which have Runge–Kutta stability property.

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