Biorthogonal systems and numerical solution of the nonlinear Volterra integro-differential equations

M. I. Berenguer[a], A. I. Garralda-Guillem[a], and M. Ruiz Galán[a]

[a]Department of Applied Mathematics, University of Granada,

Granada, Spain

maribel@ugr.es,agarral@ugr.es,mruizg@ugr.es

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Let us consider the nonlinear Volterra integro–differential equation

$$\begin{cases} y'(t) = f(t, y(t)) + \displaystyle\int_0^t K(t, s, y(s))\, ds \qquad (t \in [0,1]) \\ y(0) = y_0 \end{cases},$$

where $y_0 \in \mathbb{R}$ and $K : [0,1] \times [0,1] \times \mathbb{R} \to \mathbb{R}$ and $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ are continuous functions satisfying a Lipschitz condition at their last variables: there exist $L_f, L_K \geq 0$ such that for all $y_1, y_2 \in \mathbb{R}$ we have

$$\max_{0 \leq t \leq 1} |f(t, y_1) - f(t, y_2)| \leq L_f |y_1 - y_2|$$

and

$$\max_{0 \leq t,s \leq 1} |K(t, s, y_1) - K(t, s, y_2)| \leq L_K |y_1 - y_2|.$$

Volterra's integro-differential equations are usually difficult to solve in an analytical way. Many authors have paid attention to their numerical treatment. This work deals with obtaining a numerical method in order to approximate the solution of the nonlinear Volterra integro-differential equation. We define, following a fixed-point approach, a sequence of functions which approximate the solution of this type of equation, thanks to some properties of certain biorthogonal systems for the Banach spaces $C[0,1]$ and $C[0,1]^2$. Among the main advantages of our numerical method as opposed to the classical ones, such as collocation or quadrature, we can point out that it is not necessary to solve algebraic equation systems; furthermore, the integrals involved are immediate and therefore we do not have to require any quadrature method to calculate them. On the other hand, the method presented generalizes one developed by the authors for the linear case in a previous work. The behaviour of the method introduced will be illustrated with some examples.