An numerical approximation to the solution of an Fredholm integro-differential equation

M. I. Berenguer[a], M. V. Fernández-Muñoz[a], A. I. Garralda-Guillem[a] and M. Ruiz Galán[a]

[a]Department of Applied Mathematics, University of Granada,

Granada, Spain

maribel@ugr.es,mvfm@ugr.es,agarral@ugr.es,mruizg@ugr.es

Key words: Nonlinear Fredholm integro-differential equation, numerical methods, Schauder bases.

Several numerical methods for approximating integro-differential equations are known. These methods often transform an integro-differential equation to a linear or nonlinear system of algebraic equations which can be solved by direct or iterative methods. This work considers the specific case of the nonlinear Fredholm integro-differential equation of second kind:

$$y'(x) = g(x) + \int_a^b G(x,t,y(t))dt$$

where $g : [a,b] \to \mathbb{R}$ and $G : [a,b] \times [a,b] \times \mathbb{R} \to \mathbb{R}$ are continuous functions.

We present a method of numerical approximation for solving the above equation that uses strongly the properties of certain classical Schauder bases in the respective of Banach spaces $\mathcal{C}([a,b])$ and $\mathcal{C}([a,b] \times [a,b])$ of continuos and real-valued functions. The method is computationally attractive and some numerical examples are provided to illustrate the high accuracy of the method.