Some Generalized Iterative Methods

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For the numerical solution of partial differential equations, we have to obtain the solution of systems of linear algebraic equation of the form $Ax = b$. The most well-known direct method for solving

$$Ax = b$$

is Gaussian elimination. However, the method requires expensive computer storage and computer time, especially for problems arising from partial differential equations in three dimension.

We consider the use of iterative methods in this thesis. For linear systems involving symmetric and positive definite (SPD) matrices, the conjugate gradient (CG) method works fairly well.

For solving the linear system $Ax = b$, where the matrix A is indefinite symmetric, the methods SYMMLQ and MINRES (Paige and Saunder (1975)) work very well. Saad and Schultz generalized the method MINRES to GMRES for the nonsymmetric matrix A.

We describe the methods LQ-MINRES and QR-MINRES and show that the two methods are the same while using two different ways to factorize the tridiagonal matrix in Paige and Saunders's paper. We generalize the method QR-MINRES method for the nonsymmetric linear systems and show that the method is equivalent to the method GMRES. We also define the methods MMINRES and MSYMMQR. The difference between the method MGMRES and MMINRES is indicated. We apply the methods MMINRES and MSYMMQR to double linear system to obtain the methods LAN/MINRES and LAN/SYMMQR for nonsymmetric linear systems.