The Computation of Conformal Modules by Domain Decomposition

Nicolas Papamichael

Department of Mathematics and Statistics, University of Cyprus,

Nicosia, Cyprus

nickp@ucy.ac.cy

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Let $Q := \{\Omega;\ z_1, z_2, z_3, z_4\}$ be a (generalized) quadrilateral, consisting of a Jordan domain Ω and four specified points z_1, z_2, z_3, z_4 in counterclockwise order on its boundary, and let R_H denote a rectangle of base 1 and height H of the form $R_H := \{(\xi, \eta),\ 0 < \xi < 1,\ 0 < \eta < H\}$. Then, the conformal module $m(Q)$ of Q is the unique value of H for which Q is conformally equivalent to the rectangular quadrilateral $\{R_H;\ 0, 1, 1 + iH, iH\}$. By this it is meant that for $H = m(Q)$, and for this value only, there exists a unique conformal mapping of Ω onto R_H that takes the four specified points z_1, z_2, z_3, z_4 of Q, respectively, onto the four vertices $0, 1, 1 + iH, iH$ of R_H.

Apart from being an important domain functional from the function theoretic point of view, the conformal module of a quadrilateral is also intimately related to certain physical constants that occur in engineering applications. In particular $m(Q)$ plays a very central role in applications involving the measurement of resistance values of integrated circuit networks. For these reasons, the problem of determining $m(Q)$ is of interest both from the theoretical and the practical points of view.

In this talk we consider some of the computational and application aspects of the problem of determining $m(Q)$, and give an outline of a highly efficient domain decomposition method for computing the conformal modules of elongated quadrilaterals of the type that occur frequently in applications. (This is a report of joint work with N.S. Stylianopoulos.)