The Computation of Conformal Modules by Domain Decomposition
Nicolas Papamichael
Department of Mathematics and Statistics, University of Cyprus, Nicosia, Cyprus
nickp@ucy.ac.cy

Key words: Numerical conformal mapping; Conformal module; Domain decomposition

Let $Q := \{\Omega; z_1, z_2, z_3, z_4\}$ be a (generalized) quadrilateral, consisting of a Jordan domain $\Omega$ and four specified points $z_1, z_2, z_3, z_4$ in counterclockwise order on its boundary, and let $R_H$ denote a rectangle of base 1 and height $H$ of the form $R_H := \{(\xi, \eta), 0 < \xi < 1, 0 < \eta < H\}$. Then, the conformal module $m(Q)$ of $Q$ is the unique value of $H$ for which $Q$ is conformally equivalent to the rectangular quadrilateral $\{R_H; 0, 1, 1 + iH, iH\}$. By this it is meant that for $H = m(Q)$, and for this value only, there exists a unique conformal mapping of $\Omega$ onto $R_H$ that takes the four specified points $z_1, z_2, z_3, z_4$ of $Q$, respectively, onto the four vertices $0, 1, 1 + iH, iH$ of $R_H$.

Apart from being an important domain functional from the function theoretic point of view, the conformal module of a quadrilateral is also intimately related to certain physical constants that occur in engineering applications. In particular $m(Q)$ plays a very central role in applications involving the measurement of resistance values of integrated circuit networks. For these reasons, the problem of determining $m(Q)$ is of interest both from the theoretical and the practical points of view.

In this talk we consider some of the computational and application aspects of the problem of determining $m(Q)$, and give an outline of a highly efficient domain decomposition method for computing the conformal modules of elongated quadrilaterals of the type that occur frequently in applications. (This is a report of joint work with N.S. Stylianopoulos.)