The Singular Function Boundary Integral Method for 3-D Laplacian Problems with a Boundary Straight-Edge Singularity

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We consider the Laplace problem in a three-dimensional domain with a straight-edge singularity caused by two intersecting flat planes. The solution in the neighbourhood of such edge can be expressed as an asymptotic expansion involving the eigenpairs of the analogous two-dimensional problem in polar coordinates, which have as coefficients the so-called edge flux stress intensity functions (EFIFs). The EFIFs are functions of the axial coordinate the higher derivatives of which appear in an infinite series in the expansion.

The objective of this work is to extend the singular function boundary integral method (SFBIM) for solving the above problem and directly extracting the EFIFs. Approximating the latter by either piecewise constant or linear elements eliminates the second infinite series and allows the straightforward extension of the SFBIM. As in the case of two-dimensional problems, the solution is approximated by the leading terms of the local asymptotic solution expansion, simplified in the axial direction as explained above. These terms are also used to weight the governing harmonic equation in the Galerkin sense. The resulting discretized equations are reduced to boundary integrals by means of the divergence theorem. The Dirichlet boundary conditions are then weakly enforced by means of Lagrange multipliers. The values of the latter are calculated together with the coefficients of the EFIFs. The SFBIM appears to converge fast with the number of EFIFs, the number of the Lagrange multipliers, and the number of elements in the axial direction, leading to very accurate results.