The use of nonclassical pseudospectral method for solving nonlinear variational problems

Mohammad Maleki[a], Mansoureh Mazaheri[b]

[a]Department of Mathematics, Payame Noor University (PNU), Isfahan, Iran

[b]Department of Mathematics, Azad University, Mobarakeh, Isfahan, Iran

E-mail address: mm_maleki2005@yahoo.com

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In the large number of problems arising in analysis, mechanics, geometry, etc., it is necessary to determine the maximal or minimal of a certain functional. Problems in which it is required to investigate a function for a maximum or minimum are called variational problems.

In this paper we introduce an efficient computational method to solve variational problems for functional dependent on higher-order derivatives and functionals dependent on m functions in the following form:

(i)Extremize

$$J[x(t)] = \int_a^b F(t, x(t), x'(t), \ldots, x^{(n)}(t))dt,$$

with the given boundary conditions

$$x(a) = a_0,\ \ x'(a) = a_1,\ \ \ldots,\ \ x^{n-1}(a) = a_{n-1},$$

$$x(b) = b_0,\ \ x'(b) = b_1,\ \ \ldots,\ \ x^{n-1}(b) = b_{n-1}.$$

(ii)Extremize

$$J[x_1, x_2, \ldots, x_m] = \int_a^b F(t, x_1, x_2, \ldots, x_m, x_1', x_2', \ldots, x_m')dt,$$

with the given boundary conditions of the form

$$x_k(a) = x_k^0, \quad x_k(b) = x_k^1, \quad k = 1, 2, \ldots, m.$$

This method requires the definition of nonclassical orthogonal polynomials and collocation points (interpolation nodes) and it is applied to satisfy the Euler-Lagrange and Euler-Poisson equations (as ordinary differential equations) and it's boundary conditions at these collocation points. The application of the method to differential equations leads to an algebraic system. For the case of nonlinear differential equations, the resulting system is nonlinear system which can be solved using Newton's iterative method. Numerical examples are included to demonstrate the validity and applicability of the proposed method.