The time-dependent spectral method for solving equations of dynamic nonlinear elasticity

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The nonlinear dynamic equations of bending of thin elastic von Kármán plate are solved by means of the time-dependent spectral method. The plate is simply supported and subjected to the external constant force (compressive or stretching) λ applied at the edges of the plate. The following system of coupled two nonlinear dynamic partial differential equations is considered

$$\begin{aligned} h\rho\, w_{tt} - \rho\frac{h^3}{12}\Delta w_{tt} + hc\, w_t + D\Delta^2 w + L_\lambda w &= h[w,\psi], \text{ in } \Omega, \\ \Delta^2\psi &= -\tfrac{E}{2}[w,w], \end{aligned} \tag{1}$$

where $[w,\psi] = \partial_{11}w\partial_{22}\psi + \partial_{11}\psi\partial_{22}w - 2\partial_{12}w\partial_{12}\psi$, $w(t,x,y)$ is a deflection, $\psi(t,x,y)$ is the Airy stress potential, $w,\ \psi \in C(0,T;W^{2,2}(G))$, $\Omega = (0,T]\times G$, $G = (0,l_1)\times(0,l_2)$ is the shape of the plate. Furthermore, h, ρ, c, D and E are physical parameters of the plate and L_λ is a differential operator characterizing the external forces (compression and tension) on the edges of the plate.

The initial and boundary conditions for (1) read

$$\begin{aligned} &w(0,x,y) = u(x,y),\ w_t(0,x,y) = v(x,y) \text{ in } G, \\ &w = \Delta w = 0,\ \psi = \Delta\psi = 0 \text{ in } (0,T]\times\partial G, \end{aligned} \tag{2}$$

where $u, v \in W^{2,2}(G)$. According to the classical results of dynamic nonlinear elasticity theory the initial-boundary value problem (1), (2) has a unique solution. The solution we expand in partial sums of double Fourier's series. The global basis functions are the eigenfunctions of the Laplacian under the Dirichlet conditions. Using Galerkin's projections for (1), (2) we have

$$\mathbf{H}\mathbf{w}_N''(t) + \hat{c}\mathbf{w}_N'(t) + \mathbf{K}_1\mathbf{w}_N(t) - \mathbf{B}_\lambda\mathbf{w}_N(t) = \mathbf{A}_{1,N}(\mathbf{w}_N(t), \mathbf{K}_2^{-1}\mathbf{A}_{2,N}(\mathbf{w}_N(t),\mathbf{w}_N(t))). \tag{3}$$

Here $\mathbf{w}_N(t)$ is a vector with components which are the time-dependent Fourier coefficients, $\hat{c} = hc$, the matrices $\mathbf{H}$, $\mathbf{K}_1$, $\mathbf{K}_2$ and $\mathbf{B}_\lambda$ arise from approximations of the differential operators, and $\mathbf{A}_{1,N}$, $\mathbf{A}_{2,N}$ denote approximations of the nonlinear geometric terms in (1). The equation (3) is reduced to two first order ordinary differential equations. For solving the obtained system with the found initial conditions $\mathbf{w}_N(0)$, $\mathbf{w}_N'(0)$ the fourth order Runge-Kutta scheme is applied.