Compact Fourier Analysis for Multigrid Methods based on Block Generating Functions

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A crucial point for the efficiency of a multigrid (MG) method is the appropriate choice of its components, which allows for an efficient interplay between smoother and coarse grid correction. In many cases, this coordination can be made by means of a Local Fourier Analysis (LFA), which is an important quantitative tool for the development of new efficient MG methods. Here we consider the notion of Compact Fourier Analysis (CFA) based on block generating functions which is connected with the LFA. However, the CFA offers a more elegant and easy to survey description and a clear overview on the MG components. The principal idea of CFA is to model the MG mechanisms by means of generating functions and block symbols. The block symbol captures the behavior of the full matrices and it is absolutely simple to handle, since it is a matrix of small order.

We present the formalism and framework of the CFA approach. An important goal is to utilize the CFA for deriving MG as a direct solver, i.e. an MG cycle that will converge in just one iteration step. We give necessary and sufficient conditions that have to be fulfilled by the MG components for deriving a direct solver. We introduce general and practicable MG components that lead to MG as a direct solver. The CFA is used for calculating smoothing factors and the combined smoothing and coarse grid correction total error reduction of a twogrid step. New highly efficient MG algorithms are derived by modifying appropriately the MG components that lead to the direct solver. Numerical experiments demonstrate the theoretical results.