A numerical scheme for the modified Burgers' equation

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A finite-difference method based on third order rational approximants to the matrix-exponential term in a two-time level recurrence relation is proposed for the numerical solution of the modified Burgers' equation (MBE) given by

$$u_t + u^\mu \, u_x - \nu \, u_{xx} = 0 \,; \;\; L_0 < x < L_1 \,, \;\; t > t_0, \tag{1}$$

where μ is a positive integer with $\mu \geq 2$ - the case $\mu = 1$ corresponds to the classical Burgers' equation, $u = u\,(x,t)$ is a sufficiently often differentiable function and ν is a constant, which can be interpreted as viscosity, controlling the balance between convection and diffusion. The cases $\mu = 2$ and $\mu = 3$ will be examined. The MBE equation has the strong nonlinear aspects of the governing equation in many practical transport problems such as nonlinear waves in a medium with low-frequency pumping or absorption, ion reflection at quasi-perpendicular shocks, turbulence transport, wave processes in thermoelastic medium, transport and dispersion of pollutants in rivers and sediment transport, etc. Numerical solutions of the MBE equation can be found among others in [1, 2], etc.

The resulting nonlinear system, which is analysed for stability, is solved using an already known modified predictor-corrector scheme (see [3] and the references therein). The results arising from the experiments are compared with the relevant ones known in the bibliography.

References

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