The numerical solution of Volterra integral equation by the forward-jumping method

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As it is known, the integral equations play an important role in pure and computational mathematics. They are useful also for the numerical calculations in many problems of mechanical fluctuations in theoretical physics. Remark that investigation researches of the integral equations with variable boundaries began with Abel's works. The new beginning in the theory of integral equations is due mainly to Volterra. For the numerical solution of such equations we often use quadrature methods. In this work we use the forward-jumping method with constant coefficients and give sufficient conditions for its convergence.

Let's consider the following nonlinear Volterra type integral equation:

$$y(x) = f(x) + \int_{x_0}^{x} K(x, s, y(s))ds, \quad x \in [x_0, X]. \tag{1}$$

Sometimes this equation is referred to as Volterra-Uryson equation.

Suppose that $K(x, s, y)$ a continuous function in some closed domain G and the equation (1) has unique solution on the segment $[x_0, X]$.

Applying a quadrature method to equation (1) we have:

$$y_n = f_n + \sum_{i=1}^{n} \alpha_i K(x_n, x_i, y_i). \tag{2}$$

As it is apparent from (2), the amount of computing work for the kernel function $K(x, s, y)$ at the point x_n equals to n, and at the point x_{n+1} equals to $n + 1$. In this regard, the amount of computing work increases during the transaction from one point to another. For the numerical solution of equation (1) we suggest the forward-jumping method, which is free from the noted disadvantage:

$$\sum_{i=0}^{k-m} \alpha_i (y_{n+i} - f_{n+i}) = h \sum_{j=0}^{k} \sum_{i=0}^{k} \beta_{i,j} K(x_{n+j}, x_{n+i}, y_{n+i}).$$

We also find sufficient conditions for the convergence of the suggested method.

Remark that the investigation of the forward-jumping methods began from Kowell's work, published in 1910, which is fundamental to applied numerical solution of ordinary differential equations.