A parallel algorithm specifying determinants of ± 1 matrices

Chrysovalantis Sfyrakis and Marilena Mitrouli[a],

[a]Department of Mathematics, University of Athens

Panepistemiopolis 15784, Greece

hammer@math.uoa.gr, mmitroul@math.uoa.gr

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An interesting problem appearing in several applications is the specification of all possible values of determinants for matrices with elements ± 1. Let B be an $n \times n$ matrix with elements ± 1. It holds that (i) $det\ B$ is an integer and 2^{n-1} divides $det\ B$;

(ii) when $n \leq 6$, the only possible values for $det\ B$ are the following, and they do all occur:

n	1	2	3	4	5	6
det B	1	0,2	0,4	0,8,16	0,16,32,48	0,32,64,96,128,160

Table 1. Possible determinant values for $n \times n$ ± 1 matrices.

In this work we study the extention of the above results for matrices of higher order. For this purpose, we developed a parallel algorithm specifying first the determinants of a 7×7 matrix with elements ± 1. The basic steps of this algorithm are:

STEP1: Create all possible $128 = 2^7$ vectors with elements {-1,1}

Distribute in the available multiple processors the following tasks:

STEP2: Create all 7×7 matrices corresponding to the 7^{128} possible combinations of vectors.

STEP3: Evaluate the determinant for each matrix by applying Gaussian elimination with complete pivoting.

By executing this algorithm, we specified that the possible values for the determinant of a 7×7 matrix with elements ± 1 are

$$0,\ 64,\ 128,\ 192,\ 256,\ 320,\ 384,\ 448,\ 512,\ 576$$

and they do all occur.

The time required for the calculation of these determinants (to find each one of them at least once) was 30 minutes.

The extention of this algorithm for the computation of all possible values of the determinants of ± 1 matrices of dimension 8 that can occur, which is an open problem, is under construction.