

Nonlinear functional equations satisfied by orthogonal polynomials

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Let c be a linear functional defined by its moments $c(x^i) = c_i$ for $i = 0, 1, \dots$. We proved that the nonlinear functional equations $P(t) = c(P(x)P(x+t))$ and $P(t) = c(P(x)P(xt))$ admit polynomial solutions which are the polynomials belonging to the family of formal orthogonal polynomials with respect to a linear functional related to c . Other types of nonlinear functional equations whose solutions are formal orthogonal polynomials are also presented. Then, orthogonality with respect to a definite inner product is studied. When c is an integral functional with respect to a weight function, the preceding functional equations are nonlinear integral equations, and these results lead to new characterizations of orthogonal polynomials on the real line, on the unit circle, and, more generally, on an algebraic curve.