A Variational method for stress fields calculation in nonhomogeneous cracked materials

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Fracture mechanics deals with the study of the formation of cracks in materials, using methods of analytical mechanics to calculate the force on a crack and the material's resistance to fracture. It applies the physics of stress and strain, the theories of elasticity and plasticity, to the microscopic defects found in real materials in order to predict the mechanical failure of bodies, and a crack propagation. However, the classical methods of calculating are not generally valid in domains with complicated geometry with non homogeneous materials.

This work deals with the crack problem simulation on such non homogeneous stratified materials. It proposes a new numerical approach based on a variational formulation of the Navier-Lame equations in a two dimensional domain. We present the variational method which provides the solution in terms of displacements field in the case of a crack existence in a two dimensional plate domain Ω made of k different layers characterized by different known material properties.

The problem definition is obtained from the equilibrium equations for a plane stress and formulated in terms of stress tensors S_i in each layer Ω_i of the domain Ω. The presented problem implements the mode I deformation of the elastic non homogeneous plate with cracks, which is perfectly jointed on the interface between two consecutive layers. The plate is loaded by the opposed surface forces on the external boundary Γ_{L_i} (top and bottom) , clamped on the surface Γ_{C_i} (on the right) and free on Γ_{F_i} (on the left). The system to solve is written:

$$\left\{ \begin{array}{rcl} -\mathrm{div}\,\mathrm{S_i} = \mathbf{F}_\mathrm{i} & \text{in} & \Omega_i,\ 1 \le i \le k \\ \mathbf{u}_\mathrm{i} = 0 & \text{on} & \Gamma_{C_i},\ 1 \le i \le k \\ S_i = 0 & \text{on} & \Gamma_{F_i},\ 1 \le i \le k \\ S_i \cdot \mathbf{n} = \mathbf{G}_\mathrm{i} & \text{on} & \Gamma_{L_1},\ \ i = 1,\ i = k \end{array} \right. \tag{1}$$

In the talk, details will be given and numerical results will be shown for $k = 2$ to prove the efficiency of the method.