Rescaling Systems of Ordinary Differential Equations: Control of Stiffness and Parallel-in-Time Integration

Nabil Nassif[a], Jocelyne Erhel[b], and Noha Karam[b]

[a]Mathematics Department, American University of Beirut,
Beirut, Lebanon

[b]IRISA, Université de Rennes,
Rennes , France

nn12@aub.edu.lb,erhel@irisa.fr,@noha.makhoul@irisa.fr

Key words: Initial Value Problems, Stiff Systems, Rescaling, Sliced-time Computations, Parallel in time Integration.

Consider the first order initial value problem in which one seeks $Y : [0, T] \to \mathbb{R}^k$, such that:

$$(S) \qquad \begin{cases} \frac{dY}{dt} = F(Y), & 0 < t \leq T \leq \infty, \\ Y(0) = Y_0, \end{cases}$$

The rescaling methodology we are presenting starts with a time-slicing procedure, governed by a uniform stopping criterion that makes solving the IVP (S), that could be stiff, equivalent to solving a sequence of initial value shooting problems (S_n).
Then, a change of variables (of both the time-variable and the solution) is made:

$$\begin{cases} t = T_{n-1} + \beta_n s, \quad \beta_n > 0 \\ Y(t) = Y_{n-1} + D_n Z(s), \end{cases}$$

and yields an equivalent sequence of rescaled initial value shooting problems (S'_n) in which the time and the solution are set to zero at the onset of every slice allowing each of them to be solved through a local approach:

$$(S'_n) \qquad \begin{cases} \frac{dZ_n}{ds} = G_n(Z_n)(s)), & 0 < s \leq s_n \\ Z_n(0) = 0, \\ H[Z_n(s_n)] = 0. \end{cases}$$

The change of the time-variable uses a time-rescaling factor β_n that intends to control the growth of G_n and of its jacobian, i.e. control the stiffness of the original problem. When combined with a relevant End-Of-Slice condition, $H[Z_n(s_n)] = 0$, this technique provides the rescaled systems (S'_n) with a property of **uniform similarity** which infers for certain forms of $F(.)$, the uniform boundedness of the rescaled solutions $\{Z_n(s)\}$ on all slices. While originally intended to kill the

stiffness of the problem (S), the rescaling methodology can be also at the basis of parallel time integration.
The paper aims at presenting cases of applications of such technique and in particular its efficiency in devising parallel algorithms for time-dependent evolution problems.

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