Solving the Boltzmann equation

Robert Beauwens
Université Libre de Bruxelles, Brussels

While the analytical study of the solutions of the Boltzmann equation made nice progress, precisely this year, its numerical solution remains a challenge staying beyond current computational capabilities. We shall review in this talk the methods that have been developed recently to solve its linear version (that applies a.o. to neutron and photon transport). We shall restrict to its monoenergetic (monochomatic) version for simplicity. We shall also focus on the developments in which our department took an active part, trying to put them within the framework of a broad historical perpective.

This will cover a.o. the development of mixed hybrid discretization methods taylored to the linear transport equation. Such methods have been actively developed in the recent years for many PDEs and their transport version is based on the even and odd (angular) parity flux decomposition that had been introduced by Vladimirov in the late nineteen fifties.

This decomposition allows us to define primal and dual and thus also mixed variational formulations while the hybrid methods are characterized by the introduction of Lagrange multipliers to enforce interface continuity properties weakly rather than imposing them in a strong sense.

On the other hand, this will also cover the use of spectral expansion methods initiated for 1D geometries by the spectral analyses of Case and Mika and applied in the nineteen sixties to the solution of 1D piecewise uniform media such as cell problems in neutronics, later generalized to multidimensional geometries through the development of transverse integration based nodal codes (which transform multi-dimensional problems into sets of one-dimensional problems coupled by transverse leakage terms).

Present developments under progress concern their direct application to multidimensional geometries and we shall indicate how such generalizations are planned.