Collocation with discontinuous Hermite elements for a tumor invasion model with heterogeneous diffusion in $1+1$ dimensions

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Over the past few years, mathematical modeling for studying highly diffusive brain tumors has been well developed. Gliomas are common malignant brain tumors characterized by their aggressive diffuse invasion of brain normal tissue. The basic mathematical model used to study highly diffuse tumors, and at the same time incorporates the brain tissue heterogeneity (white and grey matter), considers a reaction-diffusion equation with a discontinuous diffusion coefficient; namely

$$\frac{\partial \bar{c}}{\partial \bar{t}} = \nabla \cdot \left(\bar{D}(\bar{\mathbf{x}}) \nabla \bar{c} \right) + \rho \bar{c} \tag{1}$$

with

$$\bar{D}(\bar{\mathbf{x}}) = \begin{cases} D_g & , \quad \bar{\mathbf{x}} \text{ in Grey Matter} \\ D_w & , \quad \bar{\mathbf{x}} \text{ in White Matter} \end{cases} . \tag{2}$$

Working towards the development of high order numerical approximation schemes, here, concentrating on the one space dimension case, we develop and study the implementation of a Hermite Collocation method with Discontinuous Hermite elements to treat first derivative discontinuities at internal interface points, as the spatial discretization method, combined with Backward (implicit) Euler (BE) time discretization schemes. Numerical experiments are included to demonstrate the performance of the method.