Galerkin and Runge–Kutta methods: Unified formulation and a posteriori error analysis

Georgios Akrivis[a], Charalambos Makridakis[b], and Ricardo H. Nochetto[c]

[a]Department of Computer Science, University of Ioannina, 451 10 Ioannina, Greece

[b]Department of Applied Mathematics, University of Crete, 714 09 Heraklion, Crete, Greece

[c]Department of Mathematics, University of Maryland, College Park, MD 20742, USA

akrivis@cs.uoi.gr, makr@tem.uoc.gr, rhn@math.umd.edu

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We consider the time discretization of initial value problems for linear parabolic equations in an abstract Hilbert space setting,

$$\begin{cases} u'(t) + Au(t) = f(t), \quad 0 < t < T, \\ u(0) = u^0, \end{cases} \tag{1}$$

by single-step schemes.

We cast Galerkin and Runge–Kutta methods into a unified formulation; the approximate solutions U are piecewise polynomials in partitions of $[0, T]$. The residual R of U, i.e., the amount by which U misses being exact solution, is in general of suboptimal order. Therefore, the straightforward approach leads to suboptimal a posteriori error estimates.

Using suitable reconstructions $\hat{U}$ of U, we derive optimal order, residual based a posteriori error estimates.

These methods yield, under appropriate compatibility conditions, nodal approximations of order higher than the global order. We derive analogous a posteriori nodal superconvergence estimates.