Extrapolation of symmetrized Runge–Kutta methods

Annie Gorgey and Robert Chan
Department of Mathematics, University of Auckland, Private Bag,
Auckland, New Zealand
a.gorgey@math.auckland.ac.nz, chan@math.auckland.ac.nz

Key words: extrapolation, order reduction, Gauss methods, stiff problems, symmetrizer

The *extrapolation* of symmetric methods of order 2 in accelerating the convergence of numerical solutions of stiff initial value problems has been investigated by Lindberg [1], Bader and Deuflhard [2]. In this paper we extend the investigation to symmetric Runge-Kutta methods of higher order. We show how to construct *symmetrizers* for arbitrary symmetric Runge-Kutta methods (see Chan [3]) that preserve the h^2-asymptotic error expansions while providing the necessary damping for stiff problems. We investigate the resulting suppression of order reduction that symmetric methods experience, thereby enabling extrapolation to be applied.

We construct symmetrizers for Gauss methods with 2 and 3 stages and study different ways of applying symmetrizers/extrapolation depending on how the numerical solution is propagated. In particular we analyse the error behaviour for the Prothero-Robinson problem in detail and report on some numerical experiments that show the merit of using symmetrizers.

References

[1] B. Lindberg, *On the smoothing and extrapolation for the trapezoidal rule*, BIT., **11**, 29 – 52, 1971.

[2] G. Bader and P. Deuflhard, *A semi-implicit midpoint rule for stiff systems of ordinary differential equations*, Numer. Math. **41**, 373 – 398, 1983.

[3] R. P. K. Chan, *Extrapolation of Runge–Kutta methods for stiff initial value problems*, Thesis submitted for the degree of Doctor of Philosophy at the University of Auckland, 1989.